Forecasting Inflation in Emerging Markets Using Phillips Curve and Alternative Time Series Models

Research Paper by A. Özlem Önder
Presentation by Zach Winship and Atharv Jagtap

VAR

Philips Curve

VECM

Box Jenkins

Decomposing Our Discussion

01 Models Used in the Paper

ARIMA, VAR, VECM, Naive

02 Introduction to Paper

Historical Context

03 Forecasting Inflation in Turkey

Why Forecasting Inflation is Important for Monetary Policy

04 Results

Comparing Inflation Forecasts

ARIMA (p, d, q)

Autoregressive Integration Moving Average

- Check for Stationarity (ADF Test)
- Transform the Series if Needed
- ACF/PACF Plots to Identify p and q
- Fit the Model

$$y_t = c + \phi_1 y_{t-1} + \ldots + \phi_p y_{t-p} + \theta_1 \varepsilon_{t-1} + \ldots + \theta_q \varepsilon_{t-q} + \varepsilon_t$$

AR (p)

MA (q)

(Gonzalez-Rivera, 2013)

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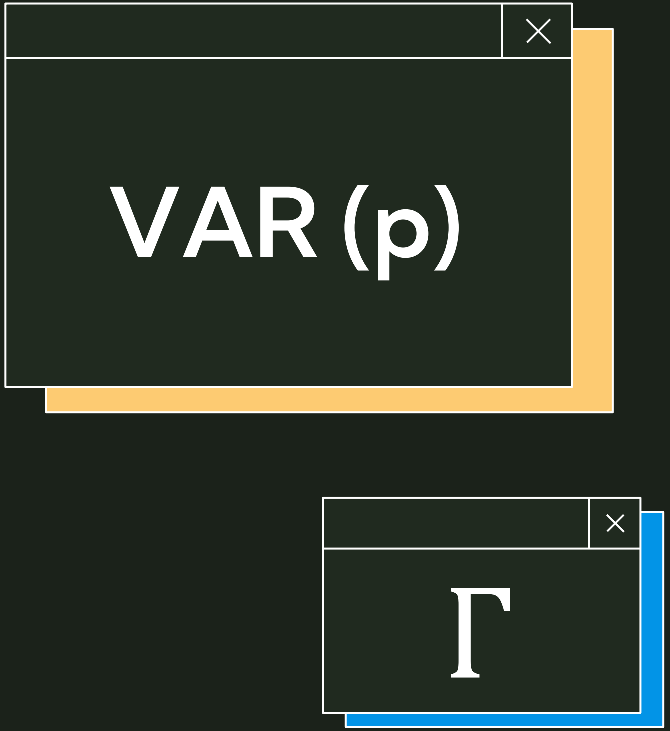
What is a Vector Autoregression?



VAR (1) Example (N =3)

$$\begin{bmatrix} M1_t & \text{Interest Rate}_t & CPI_t \\ \uparrow & \uparrow & \uparrow \\ M1_{t-1} & \text{Interest Rate}_{t-1} & CPI_{t-1} \end{bmatrix}$$

- M1 = Currency in Circulation + Demand Deposits
- CPI is used as an example for Price Level

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$$x_{t,1} = \alpha_1 + \phi_{11} x_{t-1,1} + \phi_{12} x_{t-1,2} + \phi_{13} x_{t-1,3} + \epsilon_{t,1}$$

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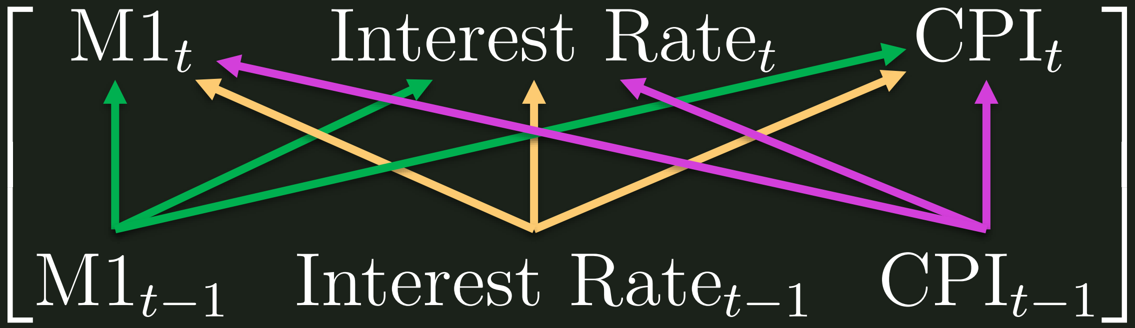

(STAT 510 – VAR Models, Penn State Online)

VAR (1) Simplied (N =3)

$$x_{t,1} = \alpha_1 + \phi_{11} x_{t-1,1} + \phi_{12} x_{t-1,2} + \phi_{13} x_{t-1,3} + \epsilon_{t,1}$$

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VAR (1) Further Simplied (N =3)

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$$\mathbf{x}_t = \boldsymbol{\alpha} + \mathbf{\Gamma}_1 \mathbf{x}_{t-1} + \boldsymbol{\epsilon}_t$$

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VAR (1) and VAR (2) and VAR (p)

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N or P can be any Positive Integer!

(STAT 510 – VAR Models, Penn State Online)

Recall Our Papers Example

$$\begin{bmatrix} \mathrm{M1}_t & \text{Interest Rate}_t & \mathrm{CPI}_t \\ \\ \mathrm{M1}_{t-1} & \text{Interest Rate}_{t-1} & \mathrm{CPI}_{t-1} \end{bmatrix}$$

P = 1 and N = 3

 # Parameters = $P \times N^2$ = 9

Imagine This Scenario

$P = 12$ and $N = 5$

Imagine This Scenario

$$
\begin{bmatrix}
x_{1,t} & x_{2,t} & x_{3,t} & x_{4,t} & x_{5,t} \\
x_{1,t-1} & x_{2,t-1} & x_{3,t-1} & x_{4,t-1} & x_{5,t-1} \\
x_{1,t-2} & x_{2,t-2} & x_{3,t-2} & x_{4,t-2} & x_{5,t-2} \\
x_{1,t-3} & x_{2,t-3} & x_{3,t-3} & x_{4,t-3} & x_{5,t-3} \\
\vdots & \vdots & \vdots & \vdots & \vdots \\
x_{1,t-12} & x_{2,t-12} & x_{3,t-12} & x_{4,t-12} & x_{5,t-12}
\end{bmatrix}
$$

P = 12 and N = 5

Imagine This Scenario

$$\begin{bmatrix} x_{1,t} & x_{2,t} & x_{3,t} & x_{4,t} & x_{5,t} \\ x_{1,t-1} & x_{2,t-1} & x_{3,t-1} & x_{4,t-1} & x_{5,t-1} \\ x_{1,t-2} & x_{2,t-2} & x_{3,t-2} & x_{4,t-2} & x_{5,t-2} \\ x_{1,t-3} & x_{2,t-3} & x_{3,t-3} & x_{4,t-3} & x_{5,t-3} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ x_{1,t-12} & x_{2,t-12} & x_{3,t-12} & x_{4,t-12} & x_{5,t-12} \end{bmatrix}$$

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P = 12 and N = 5

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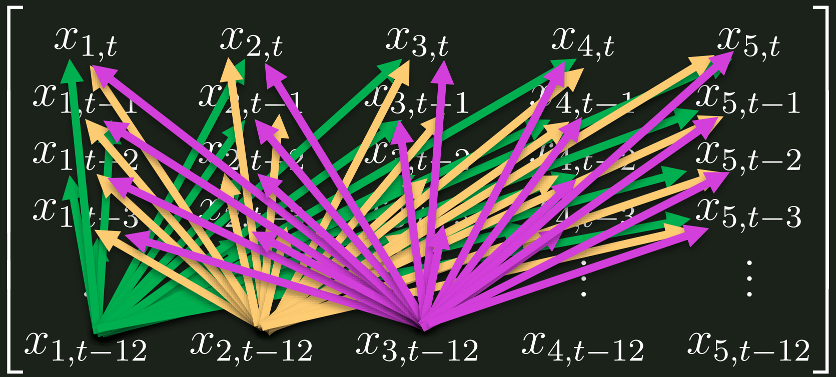

P = 12 and N = 5

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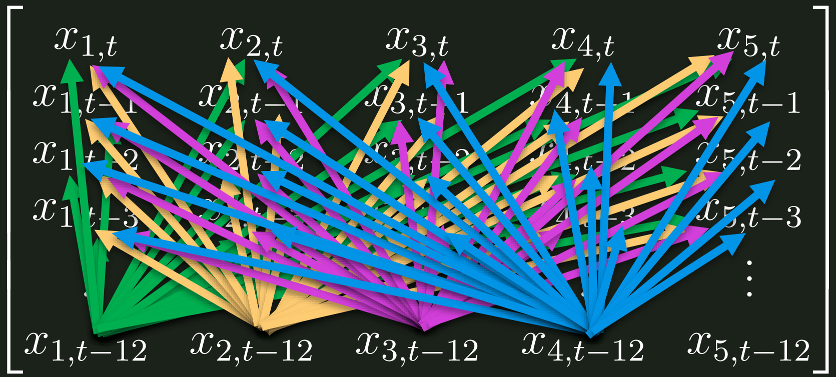

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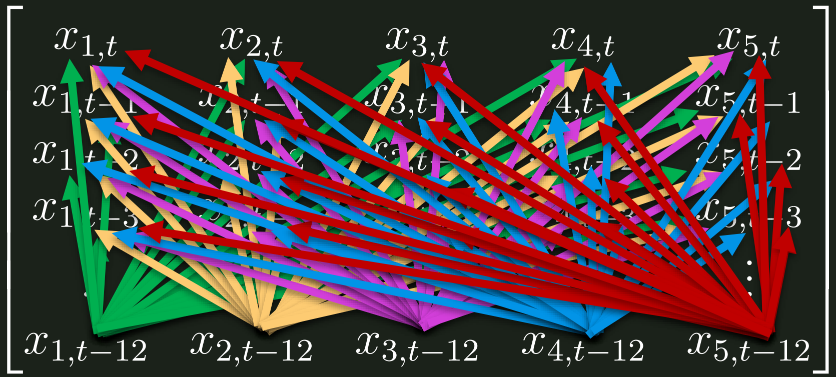

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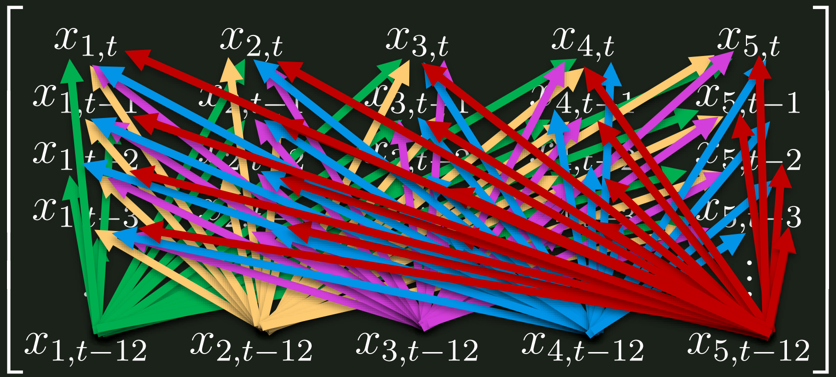
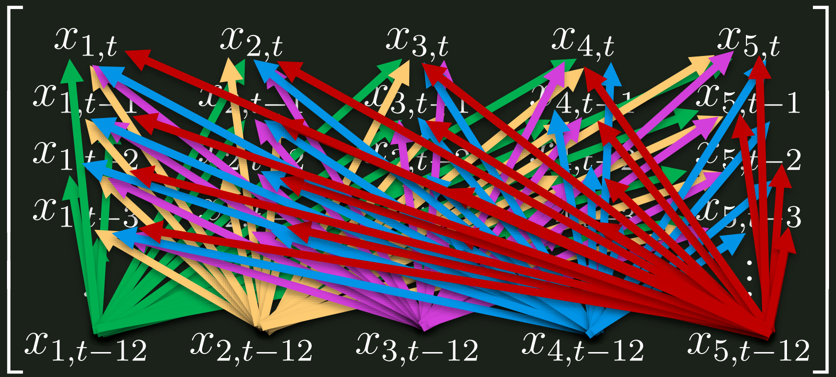


P = 12 and N = 5

 # Parameters = $P \times N^2$ = ?

Imagine This Scenario



P = 12 and N = 5

Parameters = $P \times N^2$ = 300!?!

Assuming N is Fixed. How to Choose P?

TABLE 11.2 Lag Order Selaction in VAR

VAR Lag Order Selection Criteria
Endogenous variables: GLA GRiv
Exogenous variables: C
Sample: 1975Q1 2009Q2
Included observations: 129

Lag	LogL	LR	FPE	AIC	SC	HQ
0	-569.9297	NA	24.32180	8.867127	8.911465	8.885142
1	-487.7585	160.5205	7.238637*	7.655170*	7.788185*	7.709217*
2	-487.2043	1.065406	7.636332	7.708594	7.930285	7.798671
3	-481.6932	10.42418*	7.460515	7.685165	7.995532	7.811274
4	-477.4558	7.883524	7.434896	7.681485	8.080528	7.843624
5	-476.7097	1.364911	7.822540	7.731933	8.219653	7.930104
6	-472.1276	8.240713	7.756466	7.722908	8.299304	7.957109
7	-470.3528	3.136797	8.034679	7.757408	8.422480	8.027640
8	-469.6694	1.186753	8.466835	7.808827	8.562576	8.115091

*indicates lag order selected by the criterion
LR: sequential modified LR test statistic (each test at 5% level)
FPE: Final prediction error
AIC: Akaike information criterion
SC: Schewarz information criterion
HQ: Hannan-Quinn information criterion

(Gonzalez-Rivera, 2013)

Impluse–Response Function

- Quantifies each variable's reaction to a shock from another variable
- Holds all other endogenous variables constant
- Reveals Internal dynamics of VAR model

IRF

∂

Impluse–Response Function

$$\frac{\partial x_{1,t+s}}{\partial \epsilon_{2,t}} = \frac{\Delta x_{1,t+s}}{\Delta \epsilon_{2,t}}$$

$$\begin{pmatrix} x_{t,1} \\ x_{t,2} \\ x_{t,3} \end{pmatrix} = \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{pmatrix} + \begin{pmatrix} \phi_{11} & \phi_{12} & \phi_{13} \\ \phi_{21} & \phi_{22} & \phi_{23} \\ \phi_{31} & \phi_{32} & \phi_{33} \end{pmatrix} \begin{pmatrix} x_{t-1,1} \\ x_{t-1,2} \\ x_{t-1,3} \end{pmatrix} + \begin{pmatrix} \epsilon_{t,1} \\ \epsilon_{t,2} \\ \epsilon_{t,3} \end{pmatrix}$$

(Gonzalez–Rivera, 2013)

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Note the Shock from this Example

$$\begin{pmatrix} x_{t,1} \\ x_{t,2} \\ x_{t,3} \end{pmatrix} = \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{pmatrix} + \begin{pmatrix} \phi_{11} & \phi_{12} & \phi_{13} \\ \phi_{21} & \phi_{22} & \phi_{23} \\ \phi_{31} & \phi_{32} & \phi_{33} \end{pmatrix} \begin{pmatrix} x_{t-1,1} \\ x_{t-1,2} \\ x_{t-1,3} \end{pmatrix} + \begin{pmatrix} \epsilon_{t,1} \\ \epsilon_{t,2} \\ \epsilon_{t,3} \end{pmatrix}$$

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M1 Response to r_t Shocks over Time

$$\begin{array}{r} \text{M1} \longrightarrow \\ \text{Interest Rates} \longrightarrow \\ \end{array} \begin{pmatrix} x_{t,1} \\ x_{t,2} \\ x_{t,3} \end{pmatrix} = \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{pmatrix} + \begin{pmatrix} \phi_{11} & \phi_{12} & \phi_{13} \\ \phi_{21} & \phi_{22} & \phi_{23} \\ \phi_{31} & \phi_{32} & \phi_{33} \end{pmatrix} \begin{pmatrix} x_{t-1,1} \\ x_{t-1,2} \\ x_{t-1,3} \end{pmatrix} + \begin{pmatrix} \epsilon_{t,1} \\ \epsilon_{t,2} \\ \epsilon_{t,3} \end{pmatrix}$$

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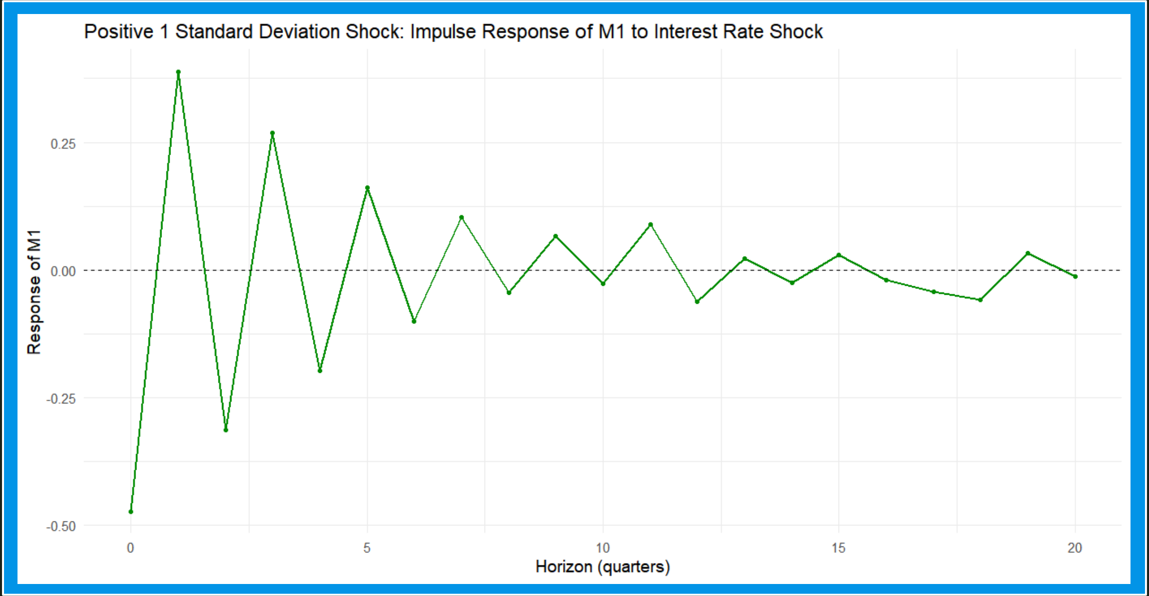
Impluse–Response Function



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Interaction Between Every Variable in VAR Model

(Gonzalez–Rivera, 2013)

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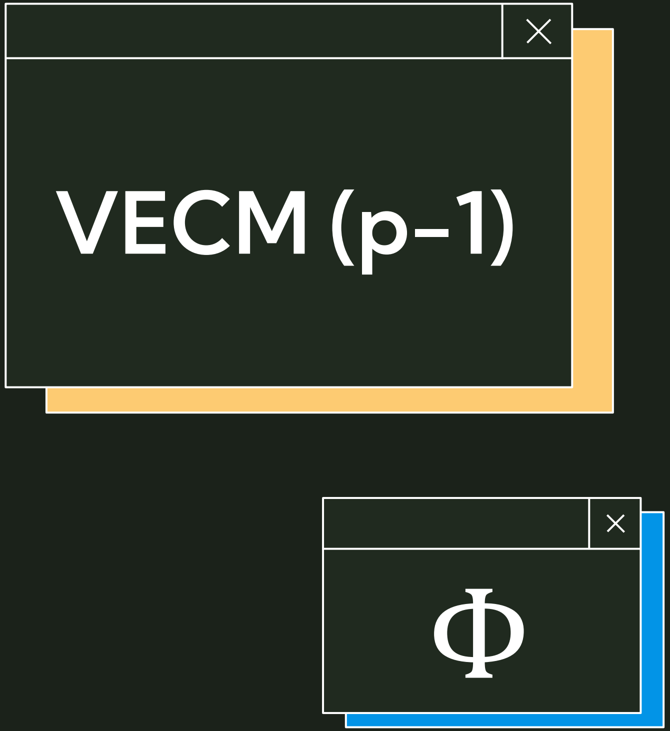
(Gonzalez–Rivera, 2013)

What is a Vector Error Correction Model?



VECM (p–1)

VECM also known as VEC

- Used when your process are Cointegrated with each other
- Very similar to VAR models
- Includes error correction mechanism
- Models in differences, not levels

β'

γ

Do I Have Cointegration?

Engle-Granger Method

- Regress Between Two Series
- ADF Test on Residuals
- Stationary Results suggest Cointegration

Johansen Test

- Multivariate Test
- Determins the Number of Cointgreation Vectors

Graphically

- Not Explicit
- But Helps to Understand Cointegration

Graphical Representation

M1
(Money Supply)

Interest Rate

CPI
(Price Proxy)

(Goldkamp, 2020)

Graphical Representation

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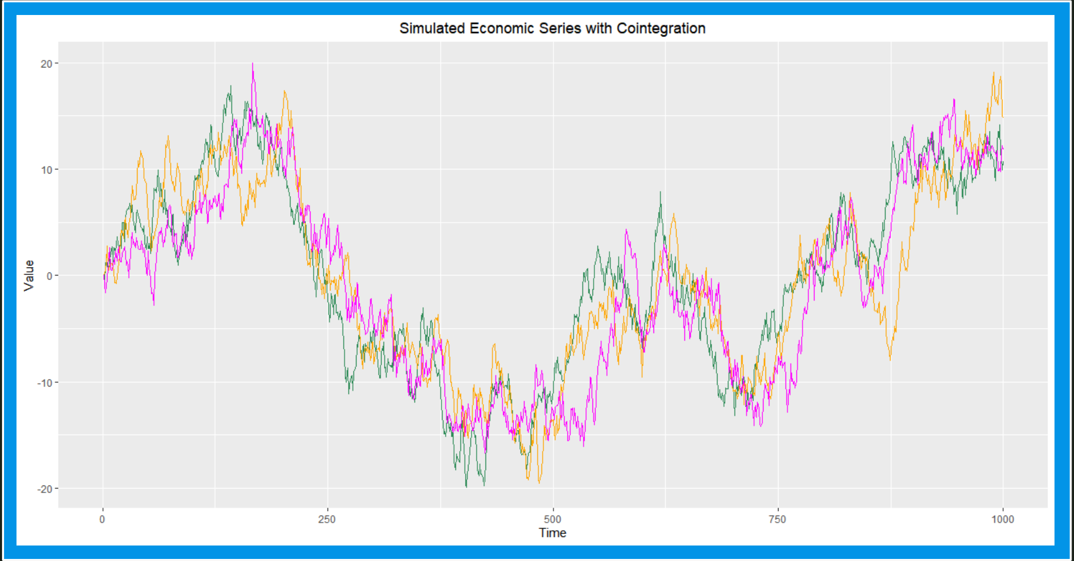
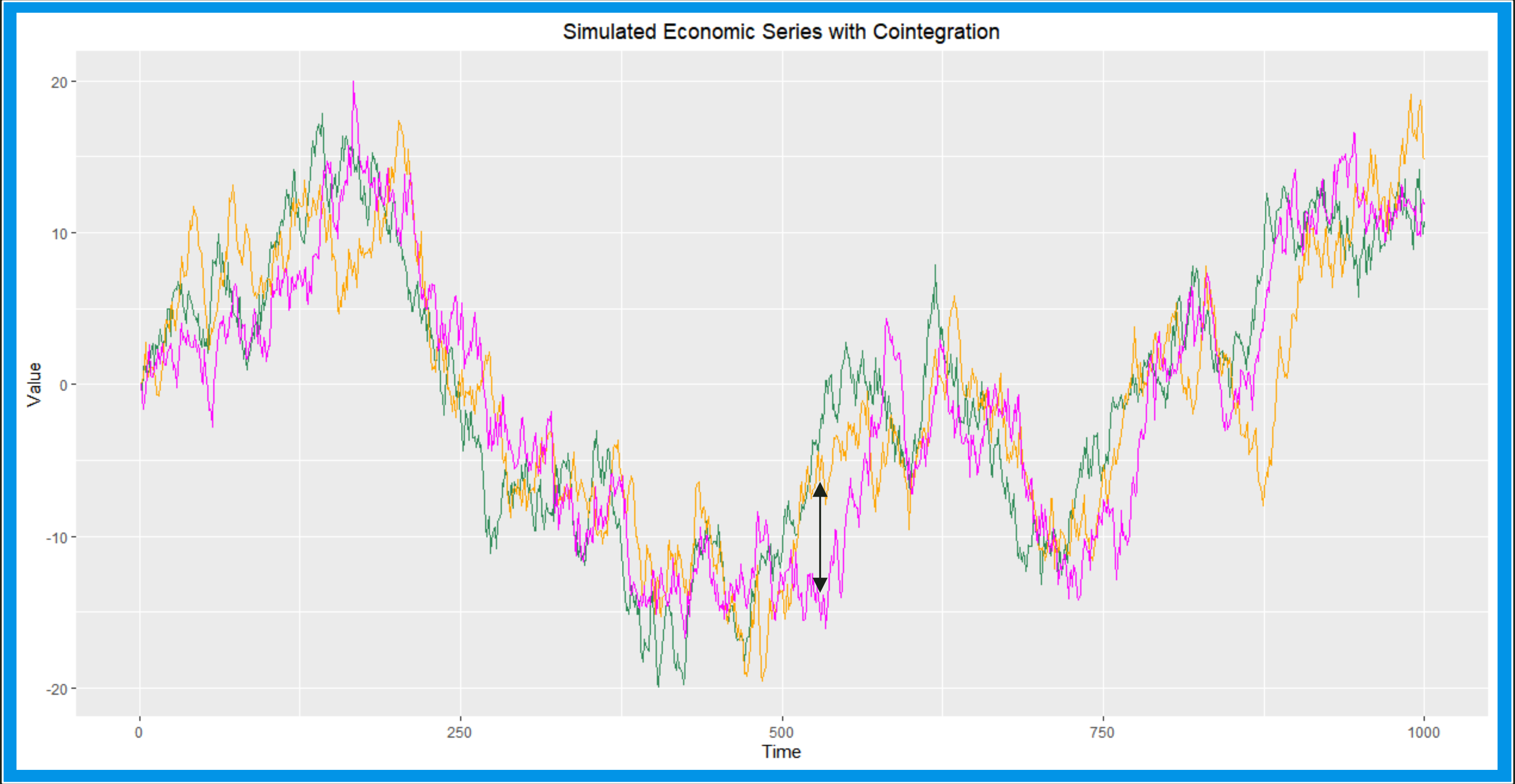


(Goldkamp, 2020)

Graphical Representation



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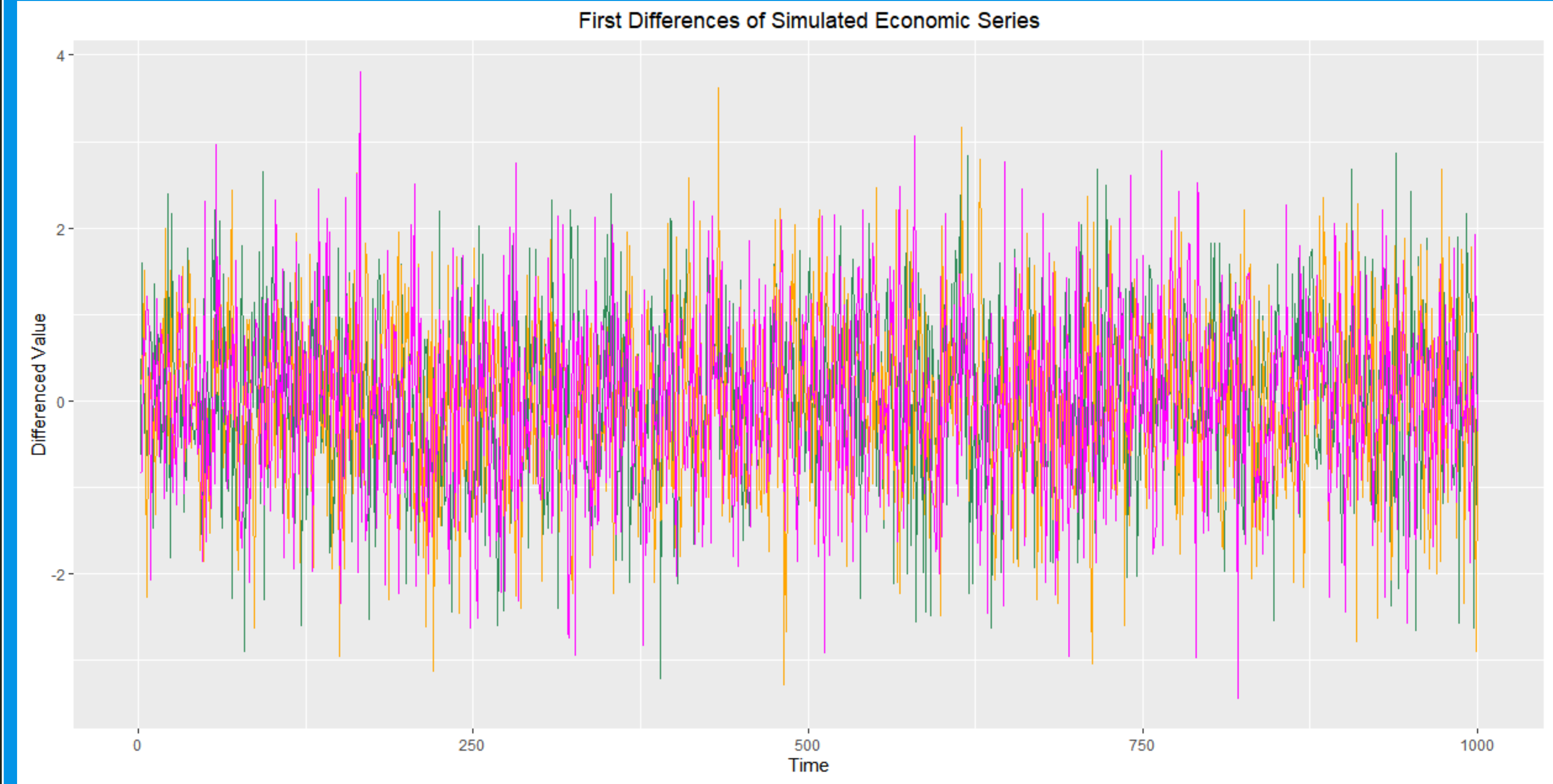

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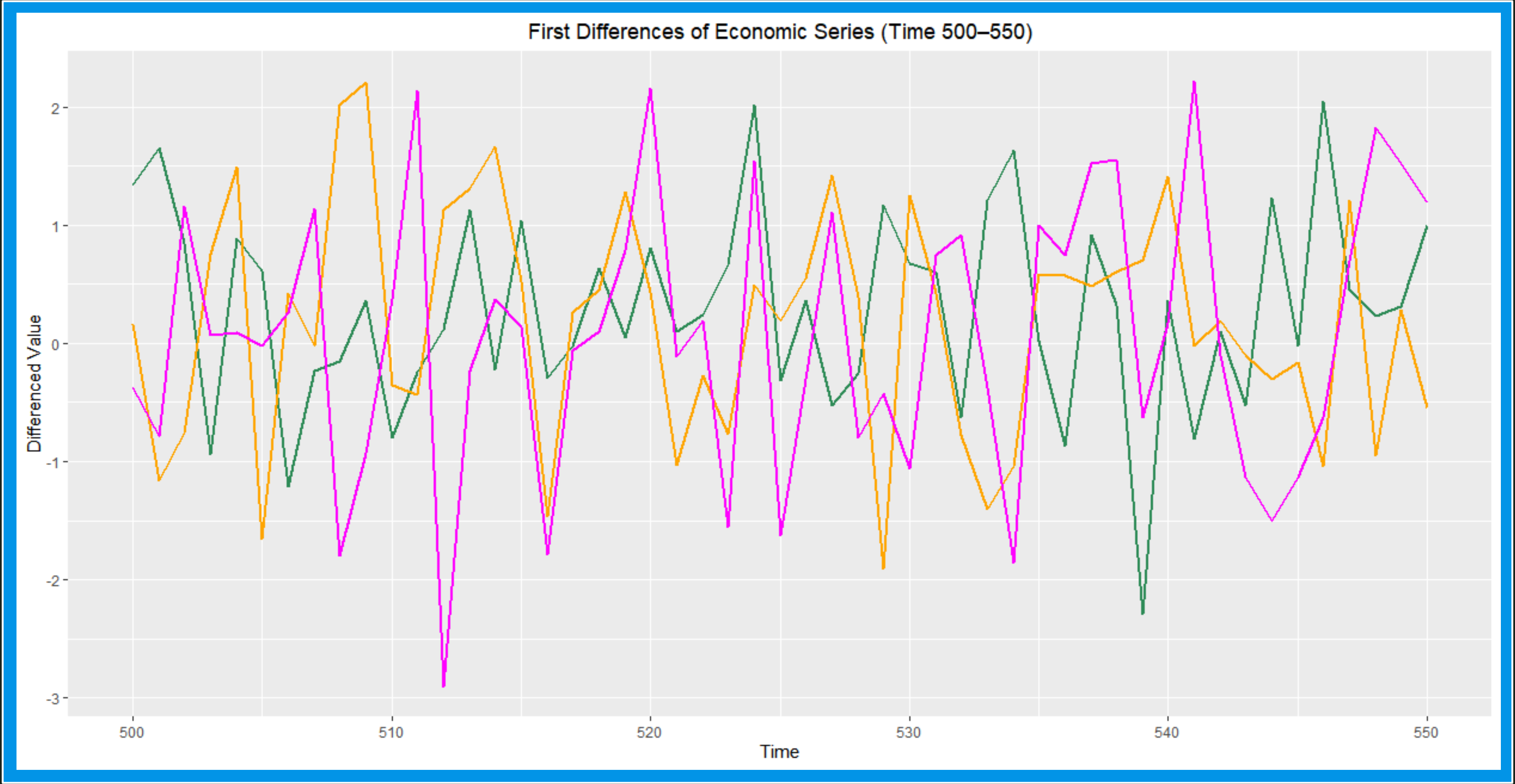
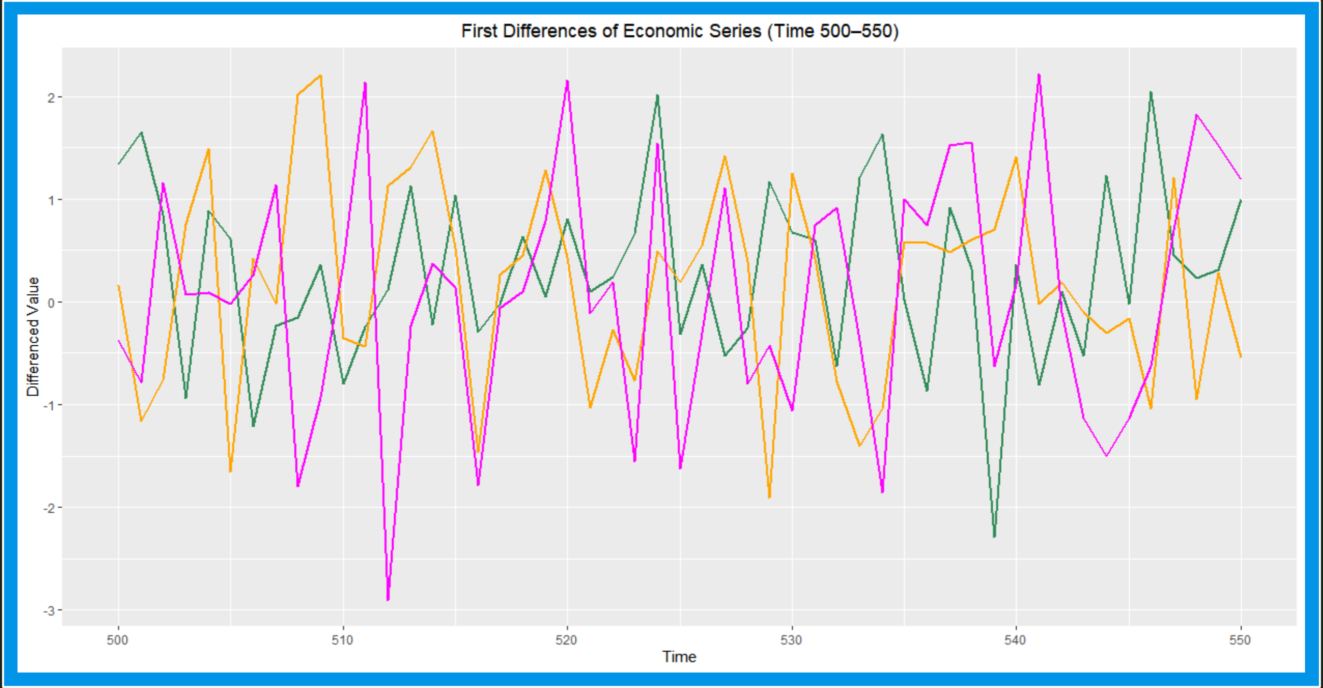


(Goldkamp, 2020)

Error Correction Term

$$ECT_t = \phi_1 x_{1,t} + \phi_2 x_{2,t} + \phi_3 x_{3,t} - \mu$$

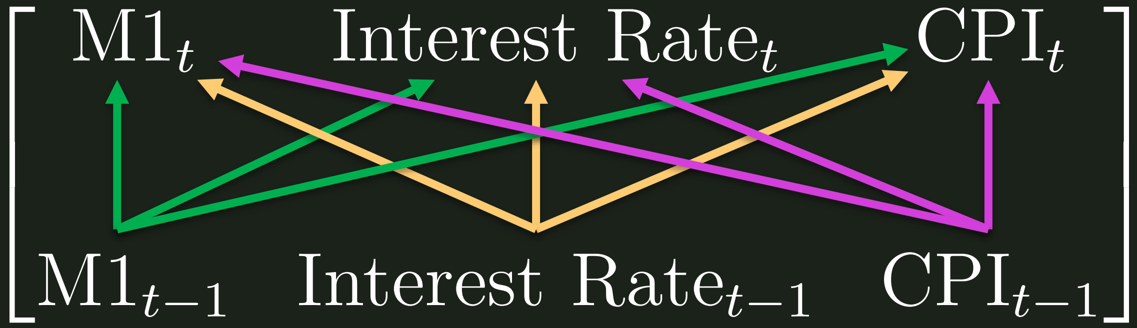


Recall the VAR (1) N = 3 Example

$$x_{t,1} = \alpha_1 + \phi_{11} x_{t-1,1} + \phi_{12} x_{t-1,2} + \phi_{13} x_{t-1,3} + \epsilon_{t,1}$$

$$x_{t,2} = \alpha_2 + \phi_{21} x_{t-1,1} + \phi_{22} x_{t-1,2} + \phi_{23} x_{t-1,3} + \epsilon_{t,2}$$

$$x_{t,3} = \alpha_3 + \phi_{31} x_{t-1,1} + \phi_{32} x_{t-1,2} + \phi_{33} x_{t-1,3} + \epsilon_{t,3}$$



(STAT 510 – VAR Models, Penn State Online)

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Take Difference $\downarrow$

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Take Difference $\downarrow$ Add ECT $ECT_t = \phi_1 x_{1,t} + \phi_2 x_{2,t} + \phi_3 x_{3,t} - \mu$

VECM Model

$$x_{t,1} = \alpha_1 + \phi_{11} x_{t-1,1} + \phi_{12} x_{t-1,2} + \phi_{13} x_{t-1,3} + \epsilon_{t,1}$$

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Take Difference ↓ Add ECT

$$\Delta x_{1,t} = \alpha_1^* + \gamma_1(\beta_1 x_{t-1,1} + \beta_2 x_{t-1,2} + \beta_3 x_{t-1,3} + \mu_1) + \phi_{11}^* \Delta x_{t-1,1} + \phi_{12}^* \Delta x_{t-1,2} + \phi_{13}^* \Delta x_{t-1,3} + \epsilon_{t,1}$$
$$\Delta x_{2,t} = \alpha_2^* + \gamma_2(\beta_1 x_{t-1,1} + \beta_2 x_{t-1,2} + \beta_3 x_{t-1,3} + \mu_2) + \phi_{21}^* \Delta x_{t-1,1} + \phi_{22}^* \Delta x_{t-1,2} + \phi_{23}^* \Delta x_{t-1,3} + \epsilon_{t,2}$$
$$\Delta x_{3,t} = \alpha_3^* + \gamma_3(\beta_1 x_{t-1,1} + \beta_2 x_{t-1,2} + \beta_3 x_{t-1,3} + \mu_3) + \phi_{31}^* \Delta x_{t-1,1} + \phi_{32}^* \Delta x_{t-1,2} + \phi_{33}^* \Delta x_{t-1,3} + \epsilon_{t,3}$$

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VECM Model Simplification

$$\Delta x_{1,t} = \alpha_1^* + \gamma_1(\beta_1 x_{t-1,1} + \beta_2 x_{t-1,2} + \beta_3 x_{t-1,3} + \mu_1) + \phi_{11}^* \Delta x_{t-1,1} + \phi_{12}^* \Delta x_{t-1,2} + \phi_{13}^* \Delta x_{t-1,3} + \epsilon_{t,1}$$
$$\Delta x_{2,t} = \alpha_2^* + \gamma_2(\beta_1 x_{t-1,1} + \beta_2 x_{t-1,2} + \beta_3 x_{t-1,3} + \mu_2) + \phi_{21}^* \Delta x_{t-1,1} + \phi_{22}^* \Delta x_{t-1,2} + \phi_{23}^* \Delta x_{t-1,3} + \epsilon_{t,2}$$
$$\Delta x_{3,t} = \alpha_3^* + \gamma_3(\beta_1 x_{t-1,1} + \beta_2 x_{t-1,2} + \beta_3 x_{t-1,3} + \mu_3) + \phi_{31}^* \Delta x_{t-1,1} + \phi_{32}^* \Delta x_{t-1,2} + \phi_{33}^* \Delta x_{t-1,3} + \epsilon_{t,3}$$

Recall the Matrix Break Down from the VAR Model

(Gonzalez-Rivera, 2013)

VECM Model Simplification

$$\Delta x_{1,t} = \alpha_1^* + \gamma_1(\beta_1 x_{t-1,1} + \beta_2 x_{t-1,2} + \beta_3 x_{t-1,3} + \mu_1) + \phi_{11}^* \Delta x_{t-1,1} + \phi_{12}^* \Delta x_{t-1,2} + \phi_{13}^* \Delta x_{t-1,3} + \epsilon_{t,1}$$
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Recall the Matrix Break Down from the VAR Model

$$\begin{pmatrix} \Delta x_{1,t} \\ \Delta x_{2,t} \\ \Delta x_{3,t} \end{pmatrix} = \begin{pmatrix} \alpha_1^* \\ \alpha_2^* \\ \alpha_3^* \end{pmatrix} + \begin{pmatrix} \gamma_1 \\ \gamma_2 \\ \gamma_3 \end{pmatrix} \begin{pmatrix} \beta_1 & \beta_2 & \beta_3 \end{pmatrix} \left(\begin{pmatrix} x_{t-1,1} \\ x_{t-1,2} \\ x_{t-1,3} \end{pmatrix} + \begin{pmatrix} \mu_1 \\ \mu_2 \\ \mu_3 \end{pmatrix} \right) + \begin{pmatrix} \phi_{11}^* & \phi_{12}^* & \phi_{13}^* \\ \phi_{21}^* & \phi_{22}^* & \phi_{23}^* \\ \phi_{31}^* & \phi_{32}^* & \phi_{33}^* \end{pmatrix} \begin{pmatrix} \Delta x_{t-1,1} \\ \Delta x_{t-1,2} \\ \Delta x_{t-1,3} \end{pmatrix} + \begin{pmatrix} \epsilon_{t,1} \\ \epsilon_{t,2} \\ \epsilon_{t,3} \end{pmatrix}$$

(Gonzalez-Rivera, 2013)

VECM Model Simplification

$$\begin{pmatrix} \Delta x_{1,t} \\ \Delta x_{2,t} \\ \Delta x_{3,t} \end{pmatrix} = \begin{pmatrix} \alpha_1^* \\ \alpha_2^* \\ \alpha_3^* \end{pmatrix} + \begin{pmatrix} \gamma_1 \\ \gamma_2 \\ \gamma_3 \end{pmatrix} \begin{pmatrix} \beta_1 & \beta_2 & \beta_3 \end{pmatrix} \left(\begin{pmatrix} x_{t-1,1} \\ x_{t-1,2} \\ x_{t-1,3} \end{pmatrix} + \begin{pmatrix} \mu_1 \\ \mu_2 \\ \mu_3 \end{pmatrix} \right) + \begin{pmatrix} \phi_{11}^* & \phi_{12}^* & \phi_{13}^* \\ \phi_{21}^* & \phi_{22}^* & \phi_{23}^* \\ \phi_{31}^* & \phi_{32}^* & \phi_{33}^* \end{pmatrix} \begin{pmatrix} \Delta x_{t-1,1} \\ \Delta x_{t-1,2} \\ \Delta x_{t-1,3} \end{pmatrix} + \begin{pmatrix} \epsilon_{t,1} \\ \epsilon_{t,2} \\ \epsilon_{t,3} \end{pmatrix}$$

$$\Delta \mathbf{x}_t = \boldsymbol{\alpha}^* + \gamma(\boldsymbol{\beta}' \mathbf{x}_{t-1} + \boldsymbol{\mu}) + \boldsymbol{\Phi}^* \Delta \mathbf{x}_{t-1} + \boldsymbol{\epsilon}_t$$

(Gonzalez-Rivera, 2013)

VECM Model Simplification

$$\begin{pmatrix} \Delta x_{1,t} \\ \Delta x_{2,t} \\ \Delta x_{3,t} \end{pmatrix} = \begin{pmatrix} \alpha_1^* \\ \alpha_2^* \\ \alpha_3^* \end{pmatrix} + \begin{pmatrix} \gamma_1 \\ \gamma_2 \\ \gamma_3 \end{pmatrix} \begin{pmatrix} \beta_1 & \beta_2 & \beta_3 \end{pmatrix} \left(\begin{pmatrix} x_{t-1,1} \\ x_{t-1,2} \\ x_{t-1,3} \end{pmatrix} + \begin{pmatrix} \mu_1 \\ \mu_2 \\ \mu_3 \end{pmatrix} \right) + \begin{pmatrix} \phi_{11}^* & \phi_{12}^* & \phi_{13}^* \\ \phi_{21}^* & \phi_{22}^* & \phi_{23}^* \\ \phi_{31}^* & \phi_{32}^* & \phi_{33}^* \end{pmatrix} \begin{pmatrix} \Delta x_{t-1,1} \\ \Delta x_{t-1,2} \\ \Delta x_{t-1,3} \end{pmatrix} + \begin{pmatrix} \epsilon_{t,1} \\ \epsilon_{t,2} \\ \epsilon_{t,3} \end{pmatrix}$$

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(Gonzalez-Rivera, 2013)

Easiest Model For Last

Naive Model

Naive Model (Level)

$$\hat{CPI}_{t+h|t} = CPI_t$$

Assumes the future value will be the same as the most recently observed value

Naive Model (Seasonal)

$$C\hat{P}I_{t+h|t} = CPI_{t+h-S(k+1)}$$

Assumes the future value will be the same as the most recent observation from the corresponding point in the previous cycle

Naive Model (Drift)

$$\hat{y}_{t+h|t} = y_t + h \times d \qquad d = \frac{y_t - y_1}{t - 1}$$

Assumes the future value will be the same as the most recent observation plus the drift of the entire series (d) times the horizon (h)

Thanks, Any Questions?